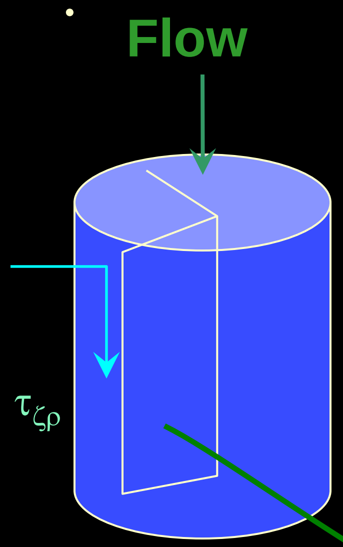
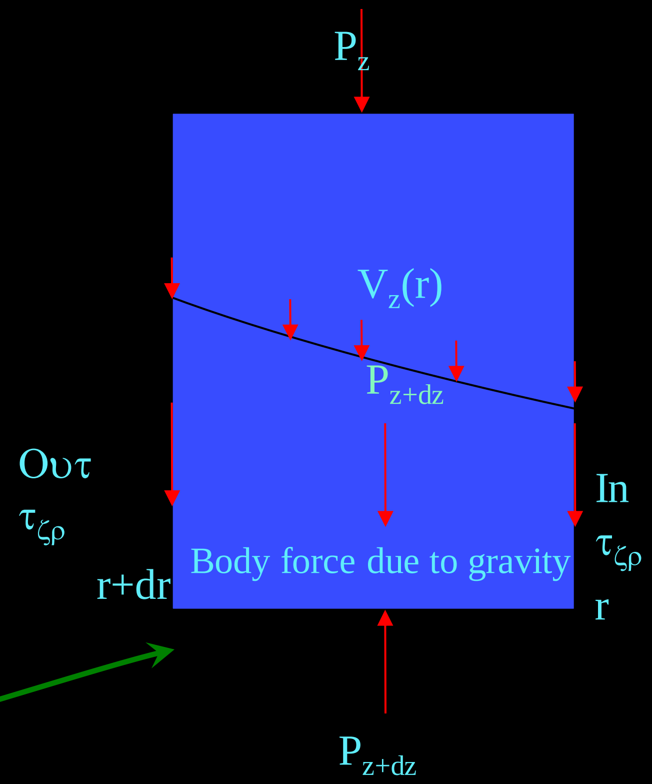
Flow In Circular Pipes

Laminar flow

Navier-Stokes equations is govern the flow field (a set of equations containing only velocity components and pressure) and can be solved exactly to obtain the **Hagen-Poiseuille relation**



If the principle of conservation of momentum is applied to a fixed volume element through which fluid is flowing and on which forces are acting, then the forces must be balanced (Newton second law)



Laminar flow Continue

Forces balance

$$\begin{bmatrix} \text{Sum of forces} \\ \text{in the z-direction} \end{bmatrix} = \begin{bmatrix} \text{Rate of change of momentum} \\ \text{in the z-direction} \end{bmatrix}$$

$$+dF_z\Big|_r = 2\pi r\,\tau_{zr}\Big|_r dz$$

$$-dF_z\Big|_{r+dr} = 2\pi(r+dr)\tau_{zr}\Big|_{r+dr} dz$$

1…Shear forces

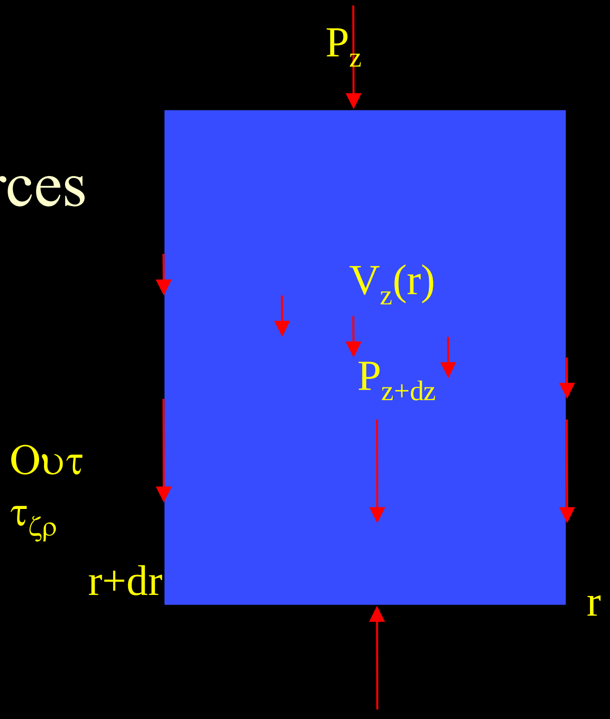
$$+p\Big|_z 2\pi r dr$$

$$-p\Big|_{z+dz} 2\pi r dr$$

2….Pressure

$$\rho g 2\pi r dr dz$$

3…..Body force



Laminar flow Continue

Momentum is

Mass*velocity **(*m*v*)**

Momentum per unit volume is

$\rho * v_z$

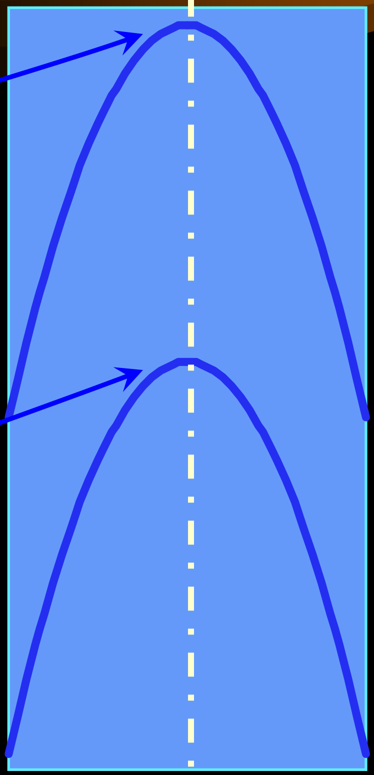
Rate of flow of momentum is

$\rho * v_z * dQ$

$dQ = v_z 2\pi r dr$

but

v_z = constant at a fixed value of r



Laminar flow

$$+\rho v_z (v 2\pi r dr)\big|_z - \rho v_z (v 2\pi r dr)\big|_{z+dz} = 0$$

Laminar flow Continue

$$2\pi r\tau_{zr}\big|_{r} dz - 2\pi(r+dr)\tau_{zr}\big|_{r+dr} dz + p\big|_{z} 2\pi r dr - p\big|_{z+dz} 2\pi r dr + \rho g 2\pi r dr dz = 0$$

$$\tau = \mu \frac{dv_z}{dr}$$

$$\Delta p = p_{z=0} - p_{z=L} + \rho g L$$

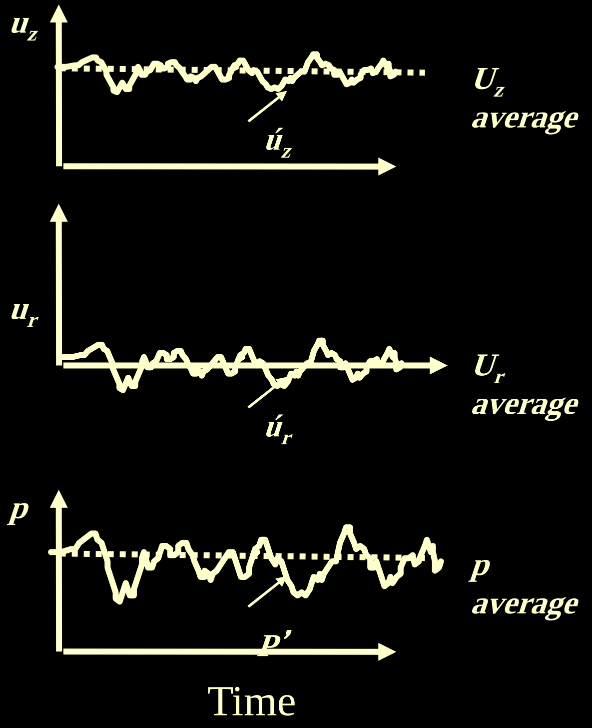
$$Q = \int_0^R 2\pi v_z dr = \frac{\pi R^4}{8\mu}\frac{\Delta p}{L}$$

Hagen-Poiseuille

Turbulent flow

- When fluid flow at higher flowrates, the streamlines are not steady and straight and the flow is not laminar. Generally, the flow field will vary in both space and time with fluctuations that comprise "turbulence
- For this case almost all terms in the Navier-Stokes equations are important and there is no simple solution

$$\Delta P = \Delta P\,(\mathrm{D}, \mu, \rho, L, U,)$$



Turbulent flow

All previous parameters involved three fundamental dimensions,

Mass, length, and time

From these parameters, three dimensionless groups can be build

$$\frac{\Delta P}{\rho U^2} = f(\mathrm{Re}, \frac{L}{D})$$

$$\mathrm{Re} = \frac{\rho U D}{\mu} = \frac{\text{inertia}}{\text{Viscous forces}}$$

Friction Factor for Laminar Turbulent flows

From forces balance and the definition of Friction Factor

$$\Delta P \times A_c = \bar{\tau} \times S \times L$$

$$\frac{A_c}{S} = r_h = \frac{1}{4} D$$

$$\bar{\tau} = \frac{\Delta P}{2L} R$$

A_c: cross section area of the pip

S: Perimeter on which *T acts (wetted perimeter)*

R_h hydraulic radius

$$f = \frac{\bar{\tau}}{1/2 \rho U^2}$$

$$f = \frac{\Delta \bar{P} R}{\rho U^2 L}$$

For Laminar flow (Hagen - Poiseuill eq)

$$Q = \frac{\pi r^4}{8\mu} \frac{\Delta P}{L}$$

$$\frac{\Delta P}{L} = \frac{8\mu U}{R^2}$$

$$f = \frac{\Delta P}{L} \frac{R}{\rho U^2} = \frac{8\mu}{\rho U R} = \frac{16}{\mathrm{Re}}$$

For Turbulent Flow

$$f = \frac{\Delta P}{L} \frac{D}{2 \rho U^2} = 0.079 \, \mathrm{Re}^{-0.25}$$

Turbulence: Flow Instability

- In turbulent flow (high Reynolds number) the force leading to stability (viscosity) is small relative to the force leading to instability (inertia).
- Any disturbance in the flow results in large scale motions superimposed on the mean flow.
- Some of the kinetic energy of the flow is transferred to these large scale motions (eddies).
- Large scale instabilities gradually lose kinetic energy to smaller scale motions.
- The kinetic energy of the smallest eddies is dissipated by viscous resistance and turned into heat. (=head loss)

Velocity Distributions

- Turbulence causes transfer of momentum from center of pipe to fluid closer to the pipe wall.
- Mixing of fluid (transfer of momentum) causes the central region of the pipe to have relatively constant velocity (compared to laminar flow)
- Close to the pipe wall eddies are smaller (size proportional to distance to the boundary)

Surface Roughness

Additional dimensionless group **ε/D** need to be characterize

Thus more than one curve on friction factor-Reynolds number plot

↓

Fanning diagram or Moody diagram

Depending on the laminar region.

If, at the lowest Reynolds numbers, the laminar portion corresponds to $f = 16/Re$ Fanning Chart

or $f = 64/Re$ Moody chart

Friction Factor for Smooth, Transition, and Rough Turbulent flow

$$f = \frac{\Delta P}{L}\frac{D}{2\rho U^2}$$

Smooth pipe, Re>3000

$$\frac{1}{\sqrt{f}} = 4.0 * \log\left[\mathrm{Re} * \sqrt{f}\right] - 0.4$$

$$f = 0.079\,\mathrm{Re}^{-0.25}$$

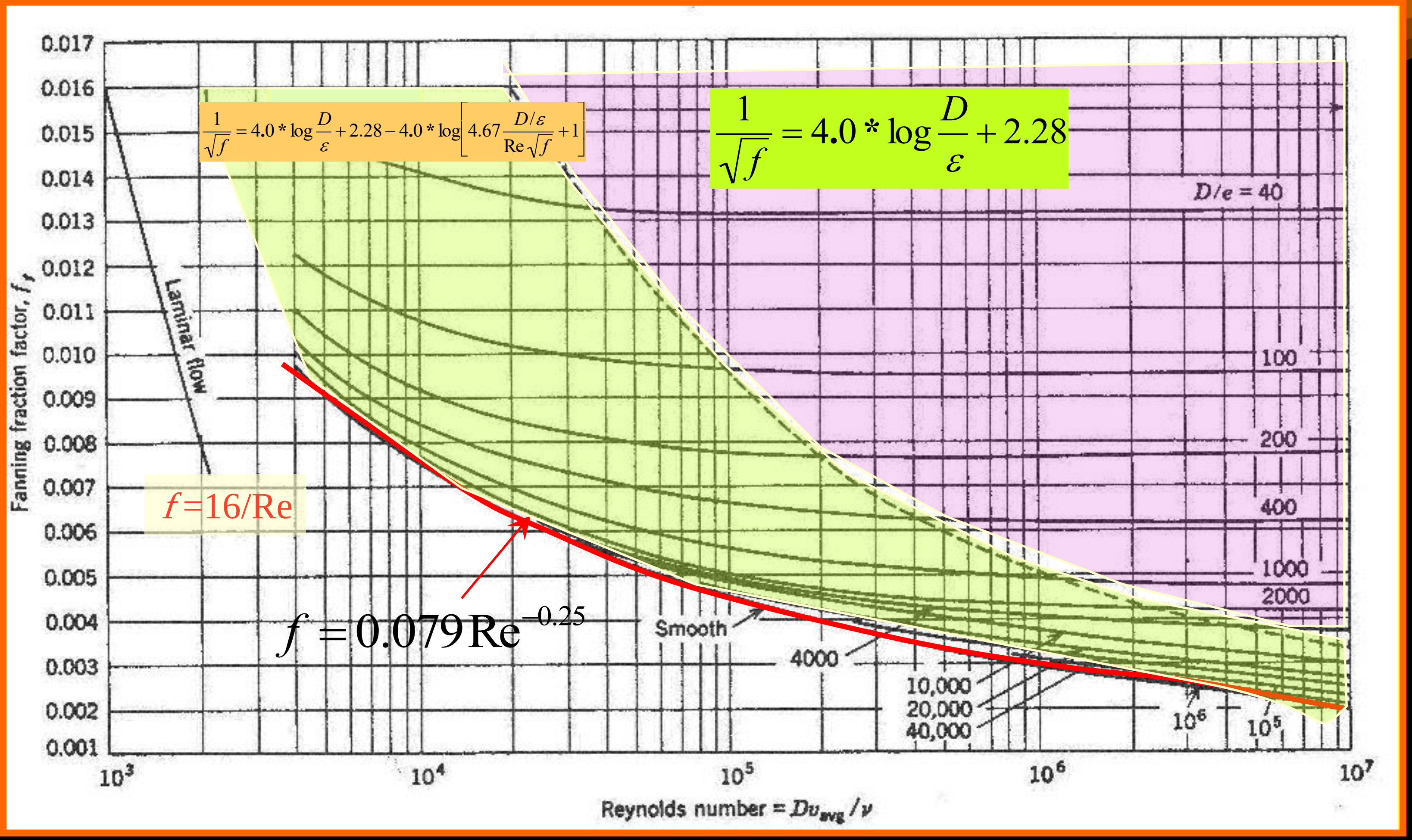
Rough pipe, [(D/ε)/(Re√*f*) <0.01]

$$\frac{1}{\sqrt{f}} = 4.0 * \log\frac{D}{\varepsilon} + 2.28$$

Transition function for both smooth and rough pipe

$$\frac{1}{\sqrt{f}} = 4.0 * \log\frac{D}{\varepsilon} + 2.28 - 4.0 * \log\left[4.67\frac{D/\varepsilon}{\mathrm{Re}\sqrt{f}} + 1\right]$$

Fanning Diagram



Pipe roughness

$\frac{\varepsilon}{D}$ Must be dimensionless!

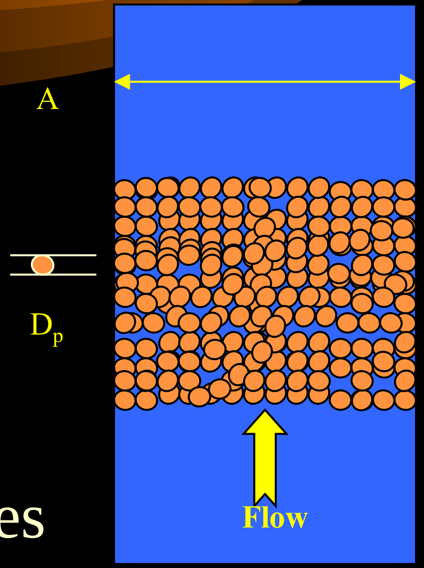
pipe material	pipe roughness ε (mm)
glass, drawn brass, copper	0.0015
commercial steel or wrought iron	0.045
asphalted cast iron	0.12
galvanized iron	0.15
cast iron	0.26
concrete	0.18-0.6
rivet steel	0.9-9.0
corrugated metal	45
PVC	0.12

Flow in a Packed pipe

The equations for empty pipe flow do not work with out considerable modification

Ergun Equation

$$f \equiv \frac{\Delta P D_p \varepsilon^3}{L \rho U_o^2 (1-\varepsilon)} = \frac{150(1-\varepsilon)\mu}{U_o D_p \rho} + 1.75$$



D_p is the particle diameter,
ε is the volume fraction that is not occupied by particles

Reynolds number for a packed bed flow as

$$Re = \frac{U_o D_p \rho}{(1-\varepsilon)\mu}$$

This equation contains the interesting behavior that the pressure drop varies as the first power of U_o for small *Re* and as U_o^2 for higher *Re*.

Energy Loss in Valves

- Function of valve type and valve position
- The complex flow path through valves can result in high head loss (of course, one of the purposes of a valve is to create head loss when it is not fully open)
- E_v are the loss in terms of velocity heads

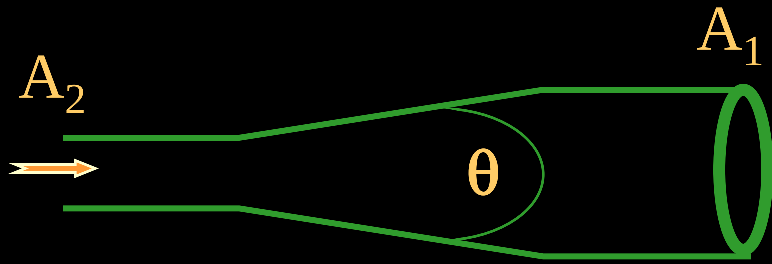
$$E_v = K\frac{U^2}{2}$$

$$h_v = \frac{\Delta p}{\rho} = K_v\frac{U^2}{2} = 2f\frac{L_{eq}}{D}\frac{U^2}{g}$$

Friction Loss Factors for valves

Valve	K	L_{eq}/D
Gate valve, wide open	0.15	7
Gate valve, 3/4 open	0.85	40
Gate valve, 1/2 open	4.4	200
Gate valve, 1/4 open	20	900
Globe valve, wide open	7.5	350

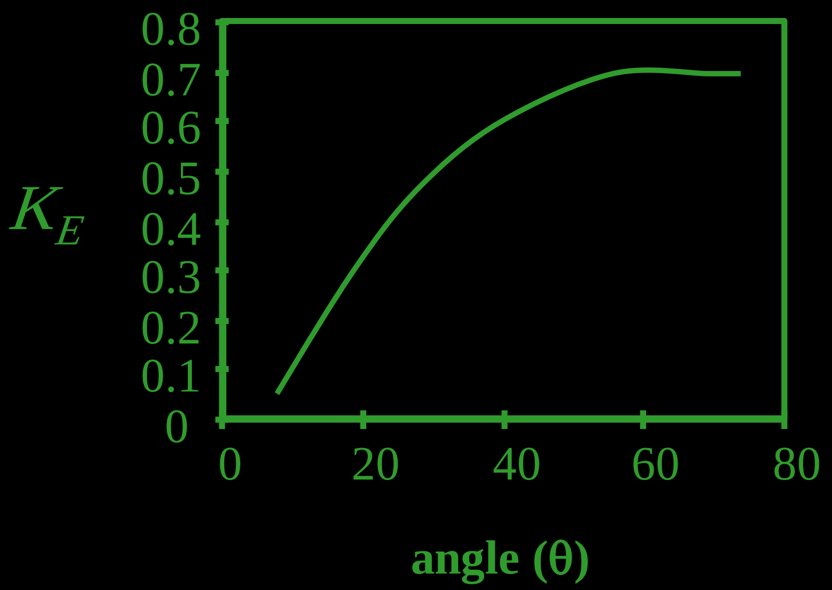
Energy Loss due to Gradual Expansion



$$E_E = K_E \frac{(U_1 - U_2)^2}{2}$$

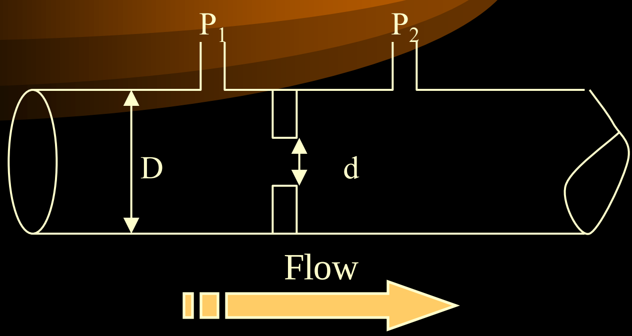
$$E_E = K_E \frac{U_2^2}{2}(\beta - 1)^2$$

$$\beta = \frac{A_2}{A_1}$$

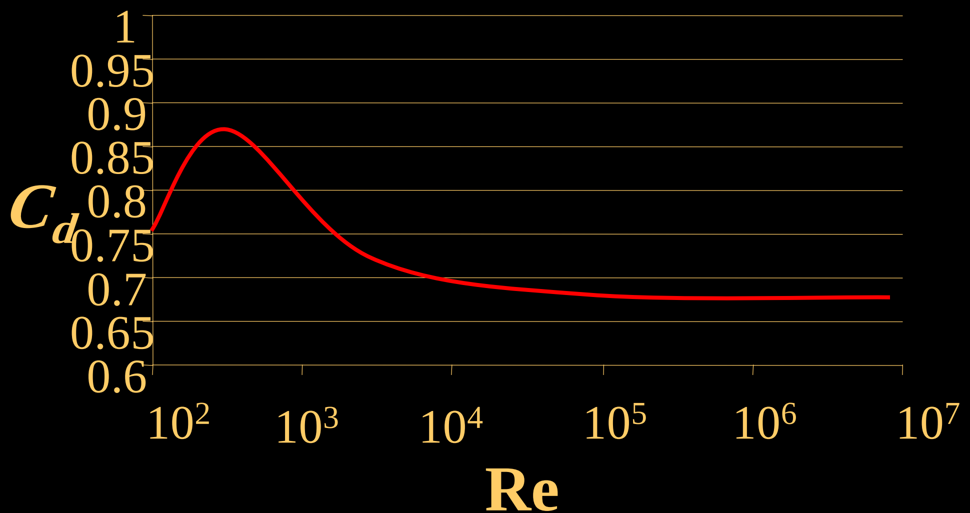


Sudden Contraction (Orifice Flowmeter)

Orifice flowmeters are used to determine a liquid or gas flowrate by measuring the differential pressure *P1-P2* across the orifice plate



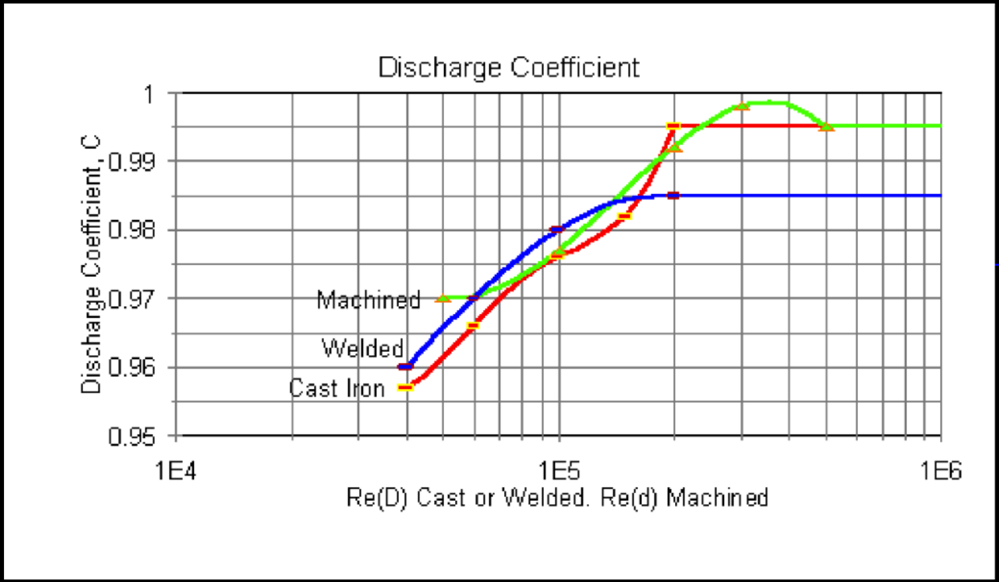
$$Q = C_d A_2 \left[\frac{2(p_1 - p_2)}{\rho(1-\beta^2)} \right]^{1/2}$$

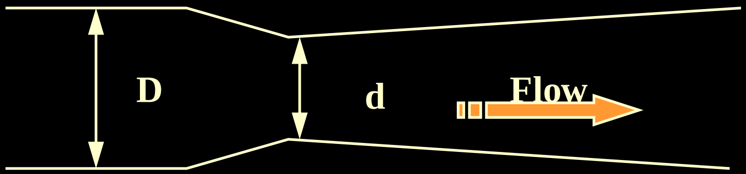


Reynolds number based on orifice diameter Re_d

Venturi Flowmeter

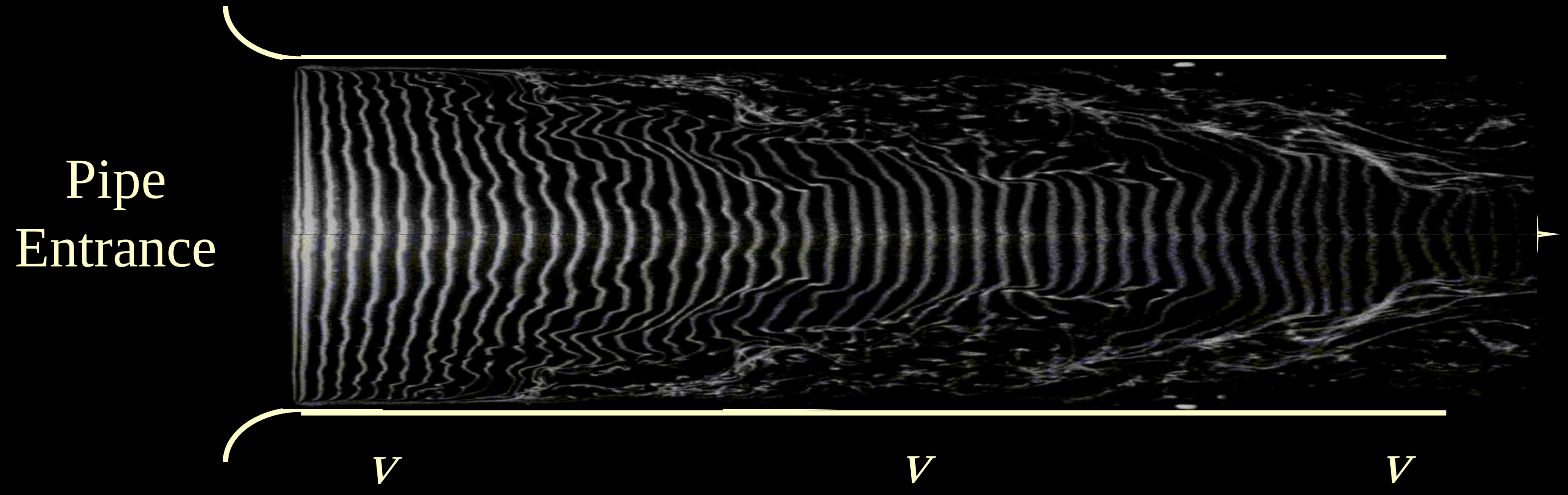
The classical Venturi tube (also known as the Herschel Venturi tube) is used to determine flowrate through a pipe. Differential pressure is the pressure difference between the pressure measured at D and at d





Boundary layer buildup in a pipe

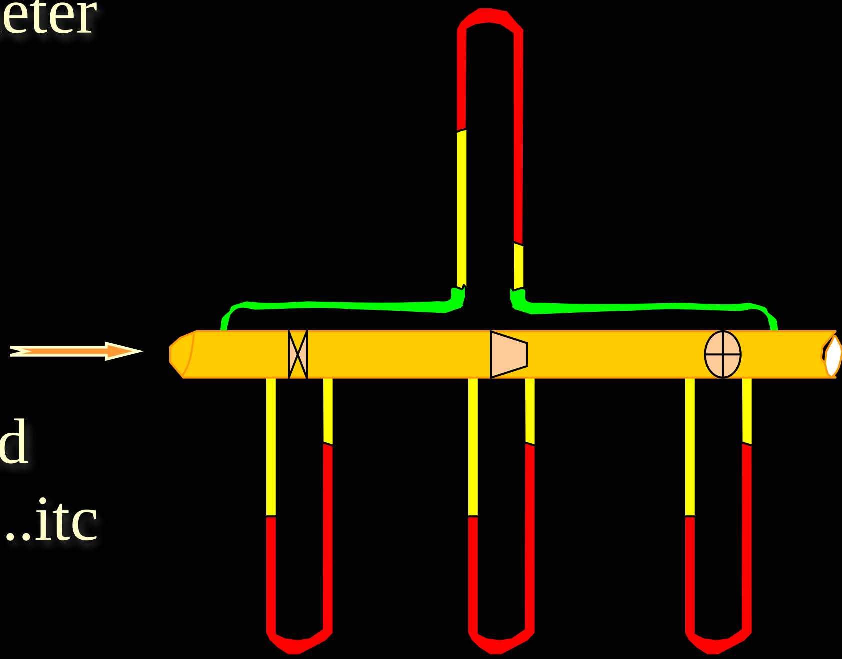
Because of the share force near the pipe wall, a boundary layer forms on the inside surface and occupies a large portion of the flow area as the distance downstream from the pipe entrance increase. At some value of this distance the boundary layer fills the flow area. The velocity profile becomes independent of the axis in the direction of flow, and the flow is said to be **fully developed**.



Pipe Flow Head Loss

(constant density fluid flows)

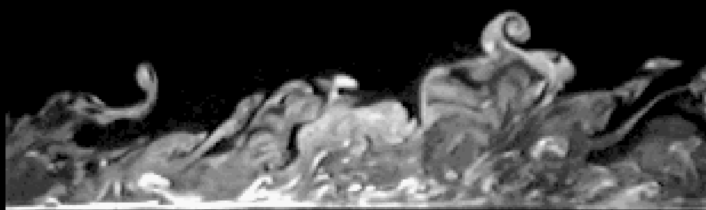
- Pipe flow head loss is
- proportional to the length of the pipe
- proportional to the square of the velocity (high Reynolds number)
- Proportional inversely with the diameter of the pipe
- increasing with surface roughness
- independent of pressure
- Total losses in the pipe system is obtained by summing individual head losses of roughness, fittings, valves ..itc



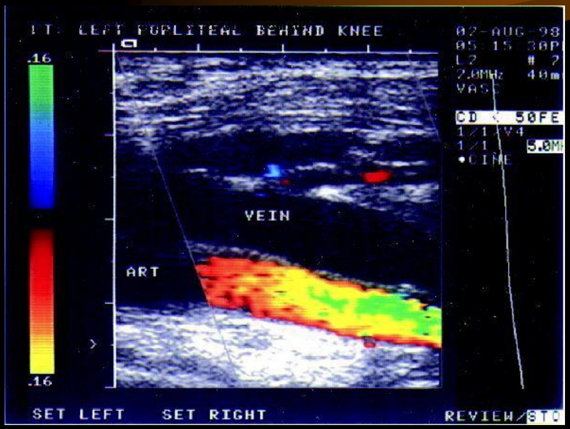
Pipe Flow Summary

- The statement of conservation of mass, momentum and energy becomes the Bernoulli equation for steady state constant density of flows.
- Dimensional analysis gives the relation between flow rate and pressure drop.
- Laminar flow losses and velocity distributions can be derived based on momentum and mass conservation to obtain exact solution named of Hagen - Poisuille
- Turbulent flow losses and velocity distributions require experimental results.
- Experiments give the relationship between the fraction factor and the Reynolds number.
- Head loss becomes minor when fluid flows at high flow rate (fraction factor is constant at high Reynolds numbers).

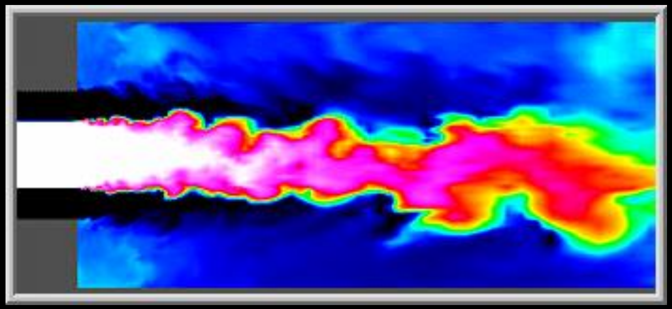
Images - Laminar/Turbulent Flows



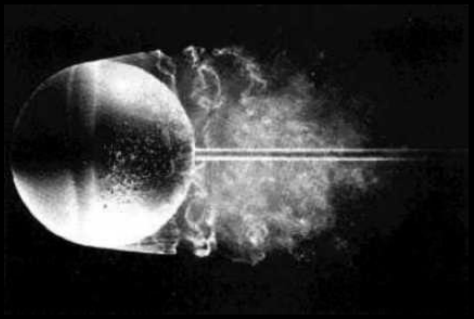
Laser - induced florescence image of an incompressible turbulent boundary layer



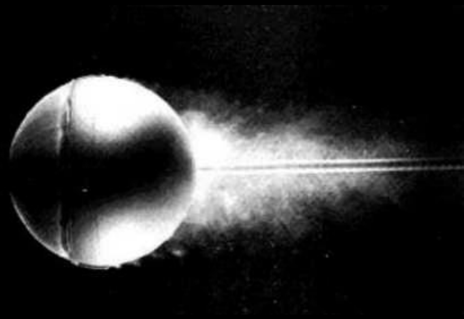
Laminar flow (Blood Flow)



Simulation of turbulent flow coming out of a tailpipe



Turbulent flow



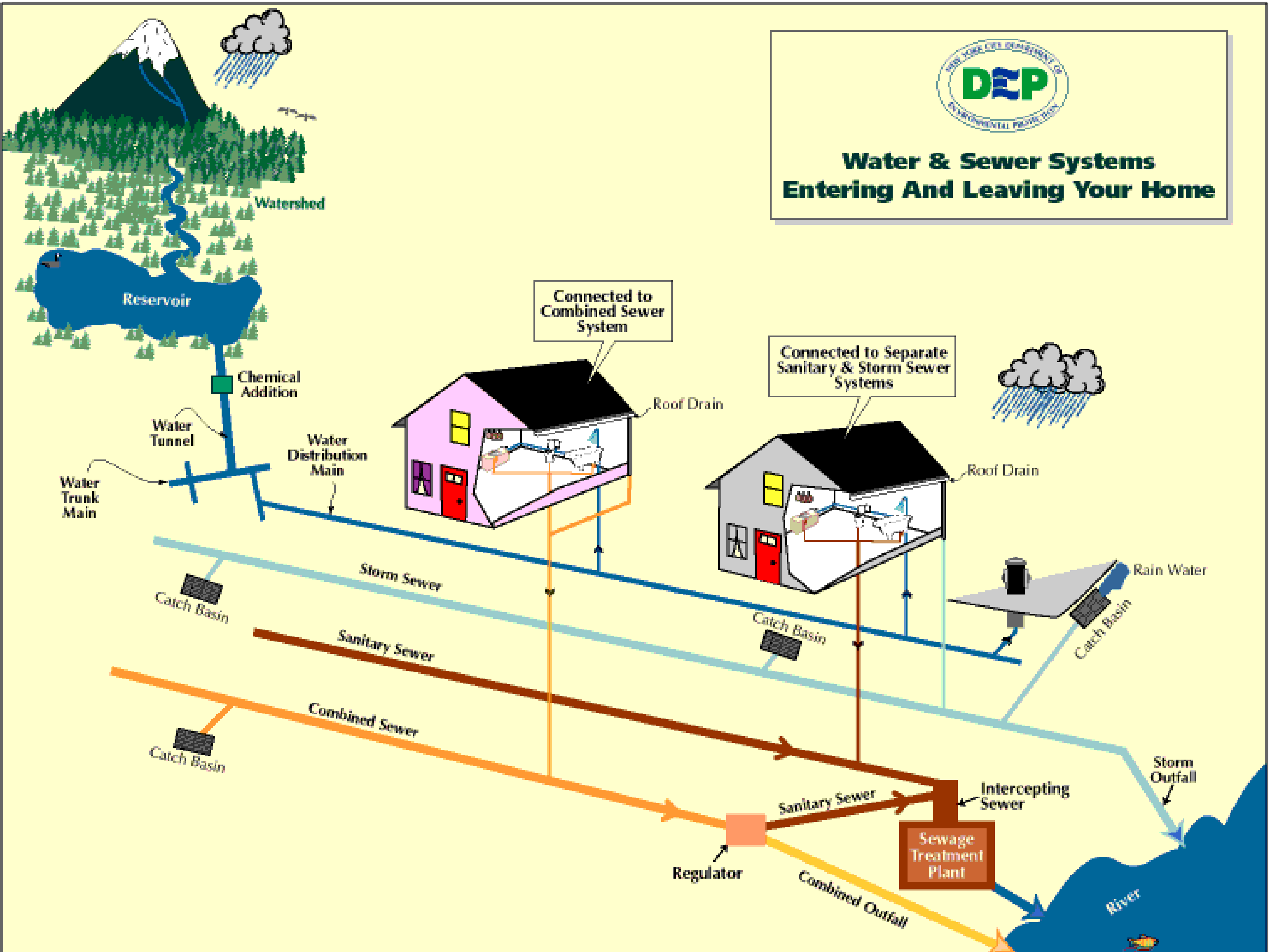
Laminar flow

http://www.engineering.uiowa.edu/~cfd/gallery/lim-turb.html

Pipes are Everywhere!

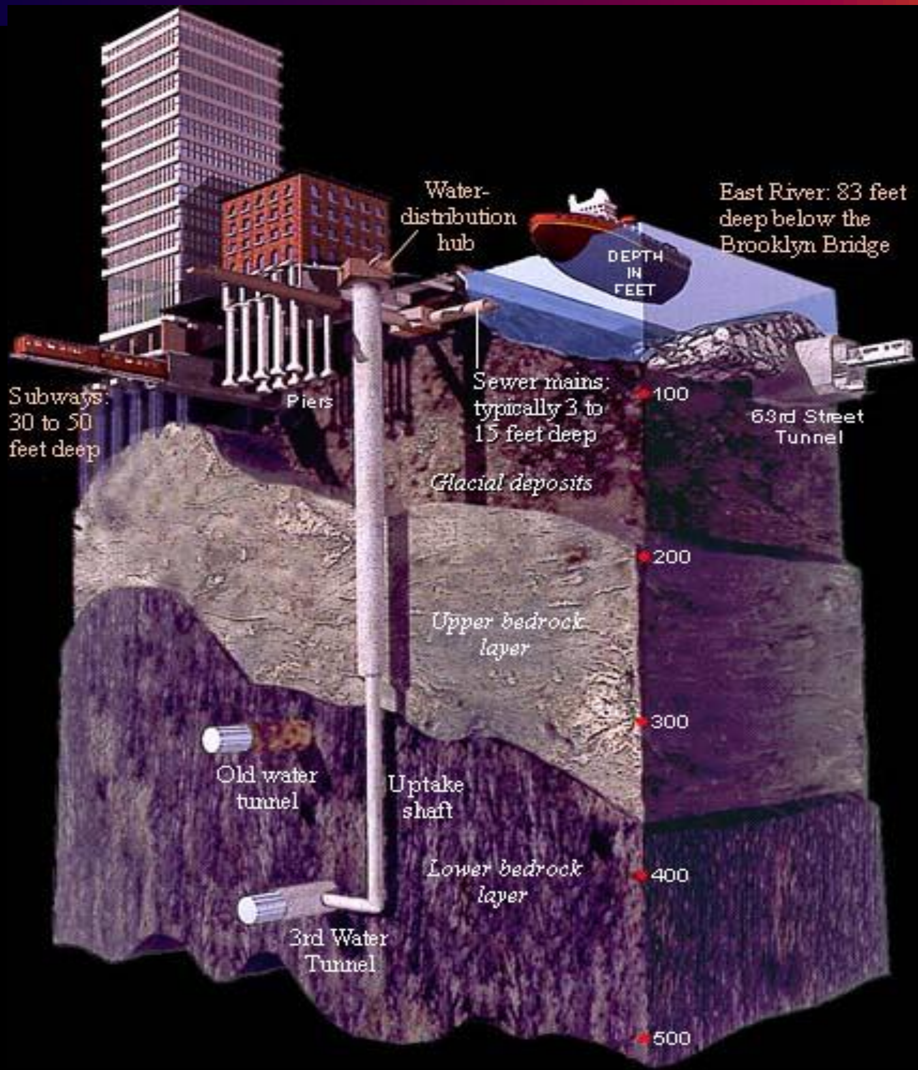
Owner: **City of Hammond, IN**
Project: **Water Main Relocation**
Pipe Size: **54"**

Pipes are Everywhere!
Drainage Pipes



Water & Sewer Systems Entering And Leaving Your Home
Watershed
Reservoir
Chemical Addition
Water Tunnel
Water Trunk Main
Water Distribution Main
Connected to Combined Sewer System
Connected to Separate Sanitary & Storm Sewer Systems
Roof Drain
Roof Drain
Rain Water
Storm Sewer
Catch Basin
Catch Basin
Catch Basin
Catch Basin
Sanitary Sewer
Combined Sewer
Sanitary Sewer
Intercepting Sewer
Sewage Treatment Plant
Regulator
Combined Outfall
Storm Outfall
River

Pipes are Everywhere!
Water Mains

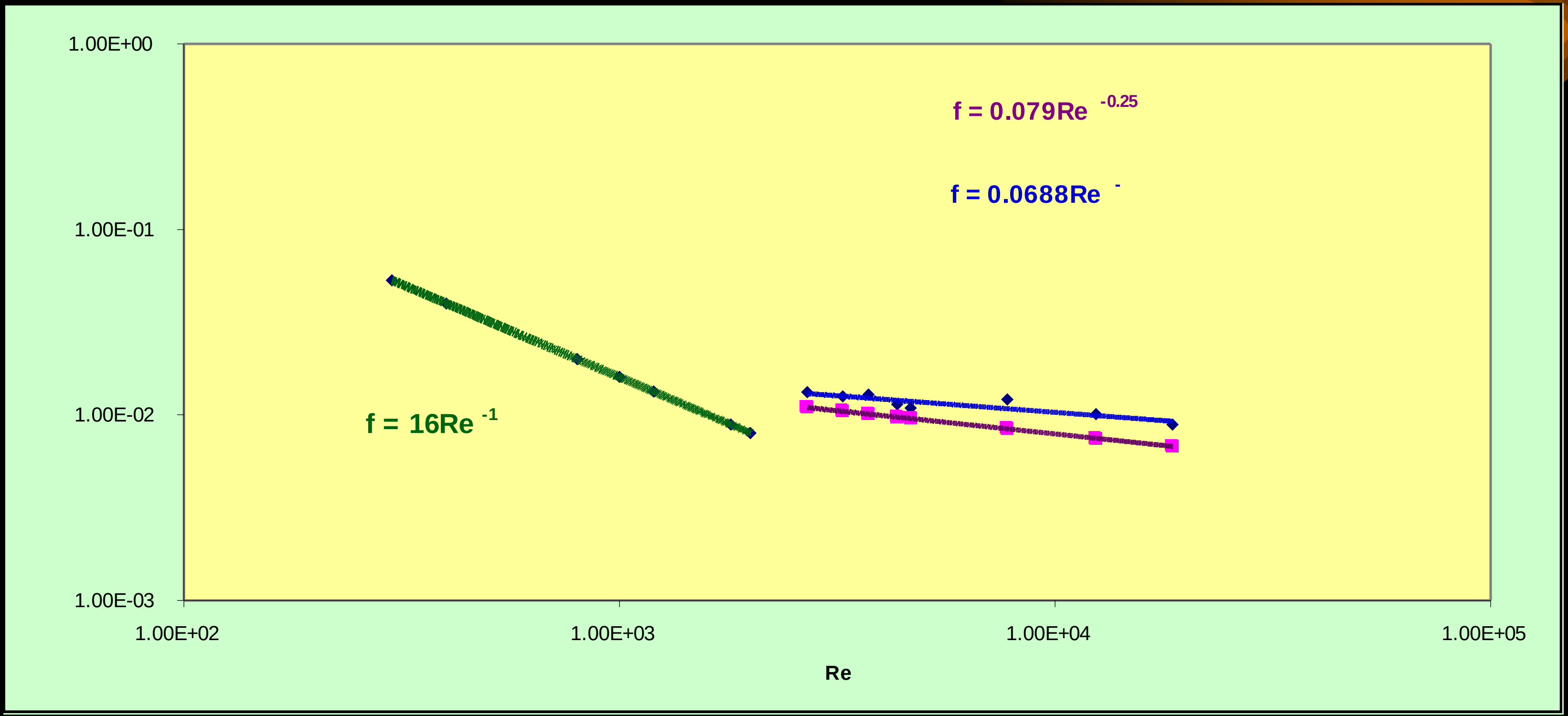


D (m)	area (m)^2	Flowrate (cc/min)	Flowrate (cc/sec)	Flowrate m^3/sec	velosity (m/sec)	Re	Presure d in(water)	Presure d m	Presure dr Pa(N/M2)	DP/L	Friction F f exp (P1)	Friction Fa f equ
0.0068326	3.66806E-05	300	5	0.000005	0.1363118	9.31E+02	0.80	2.00E-02	196.0000	1.05E+02	0.01980	0.01430
0.0068326	3.66806E-05	600	10	0.00001	0.2726235	1.86E+03	1.90	4.75E-02	465.5000	2.50E+02	0.01176	0.01203
0.0068326	3.66806E-05	870	14.5	0.0000145	0.3953041	2.70E+03	4.50	1.13E-01	1102.5000	5.93E+02	0.01324	0.01096
0.0068326	3.66806E-05	1050	17.5	0.0000175	0.4770912	3.26E+03	6.22	1.56E-01	1523.9000	8.19E+02	0.01257	0.01046
0.0068326	3.66806E-05	1200	20	0.00002	0.5452471	3.73E+03	8.30	2.08E-01	2033.5000	1.09E+03	0.01284	0.01011
0.0068326	3.66806E-05	1400	23.333333	2.333E-05	0.6361216	4.35E+03	10.00	2.50E-01	2450.0000	1.32E+03	0.01137	0.00973
0.0068326	3.66806E-05	1500	25	0.000025	0.6815589	4.66E+03	11.00	2.75E-01	2695.0000	1.45E+03	0.01089	0.00956
0.0068326	3.66806E-05	2500	41.666667	4.167E-05	1.1359314	7.76E+03	34.00	8.50E-01	8330.0000	4.48E+03	0.01212	0.00842
0.0068326	3.66806E-05	4000	66.666667	6.667E-05	1.8174903	1.24E+04	72.30	1.81E+00	17713.5000	9.52E+03	0.01007	0.00748
0.0068326	3.66806E-05	6000	100	0.0001	2.7262354	1.86E+04	143.00	3.58E+00	35035.0000	1.88E+04	0.00885	0.00676

$$\Delta P = h_{mano.reading} * (\rho_{liquid} - \rho_{water}) * g$$

$$\mathrm{Re} = \frac{\rho D V}{\mu}$$

$$f = \frac{\Delta P}{L} \frac{D}{2 \rho U^2}$$



1.00E+00
1.00E-01
1.00E-02
1.00E-03
f = 0.079Re -0.25
f = 0.0688Re -
f = 16Re -1
1.00E+02
1.00E+03
1.00E+04
1.00E+05
Re